Behaviours of Multivibrator near Instability Point

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Summary: Logarithmic divergence of oscillation period and power law increase of its fluctuation with critical index of -1 are induced in multivibrator by destabilizing the oscillation through externally added variable resistor. They are analyzed under assumption that the charge fluctuates in the condencer which decides the time constant of the circuit. The standard deviation of the charge fluctuation is estimated from the data.

Key Words: Multivibrator, Critical Phenomena, Phase Transition

1. Introduction

The oscillatory-nonoscillatory transitions in sine (or continuous) wave oscillators have been well investigated as fairly suitable theme of phase transition in systems far form equilibrum[1,2,3,4,5]. The transition is induced by controlling the quantities of positive feedback in the oscillators. The oscillating state is assigned as ordered phase and the nonoscillating state as disordered phase. Both states are separated by critical quantity of positive feedback. Many characteristic phenomena near critial (or instability) point have been observed such as critical fluctuation[1], critical slowing down[2] or irreversible circulation of fluctuation[3]. The effect of external noise on oscillation threshold has been investigated in parametric oscillator[4] and the analogy of transition with the phase transition in equilibrium systems has been disussed using Ginzburg-Landau equation[5]. Owing to these works the transition phenomena in the sine wave oscillators can now be described in terms of stochastic processes and of phase transitions.

On the other hand different type oscillators are present from above mentioned sine wave oscillator, that is, the discontinuous wave oscillators such as multivibrator, blocking oscillator or saw-tooth oscillator. Different from the sine wave oscillators the oscillation continues through repetition of "on" and "off" of the active elements such as transitors in the circuit of these oscillators. The dynamics are essentially described by van der Pol equation with large gain limit in the discontinuous wave oscillator, while they are described by van der Pol equation with small gain in the sine wave oscillators[6]. Accordingly we can expect different type transition phenomena in such oscillators. The examples have already been reported by the author[7,8] and several problems have also been discussed. Not the oscillation amplitude but the oscillation period plays the important role as pointed out by the author, however the analyses are incomplete because the fluctuaion of period has not been discussed quantitatively. We find the example in which the divergence of period and of its fluctuation can be analyzed theoretically. It is modified multivibrator circuit and the instability phenomena will be reported in this article. In the next section the experimental data and their analyses are mentioned. In the last section concluding remarks and their technical applicability will be discussed.

2. Experimental Results and Their Analyses

2.1 Experimental Results

The multivibrator circuit used in this experiment is represented in Fig.1. This is usual type circuit except the added variable resistor R which is utilized to induce the instability in multivibrator. The dymanics of the circuit are explained qualitatively below: initially we assume $V_- = 0$ V and $V_+ \gtrsim 0$ V (point A in Fig. 2) without loss of generality. If the gain of operational amplifier is infinity, the output voltage V_o saturates to V_s ($\cong$source bias voltage). At this instance V_+ is set to be $R_2V_s/(R_1+R_2)$ and V_- becomes to increase toward $RV_s/(R+r)$ with appropriate time constant. When V_- becomes slightly higher than $V_+ = R_2V_s/(R_1+R_2)$, V_o turns out to be $-V_s$. The same action repeats, and the oscillation continues as represented in Fig.2.

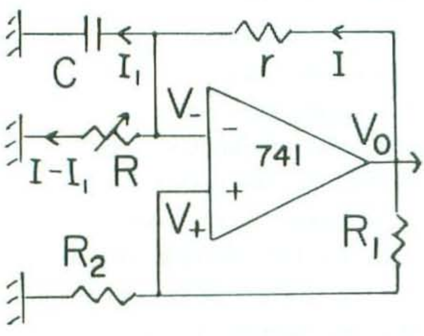

Fig.1 Muliivibrator circuit used in this experiment.

Operational amplifier is 741. The variable resistor R is used to induce the instabilty in the circuit. The parameters are $R_1 = 1M\Omega$, $R_2 = 75k\Omega$, $r = 51k\Omega$ and $C = 0.1\mu F$, respectively. V_- and V_+ are inverted and noninverted input bias voltage to operational amplifier, respectively. V_o is output bias voltage. The currents are designated as I, I_1 and $I\text{-}I_1$ through r, C and R, respectively.

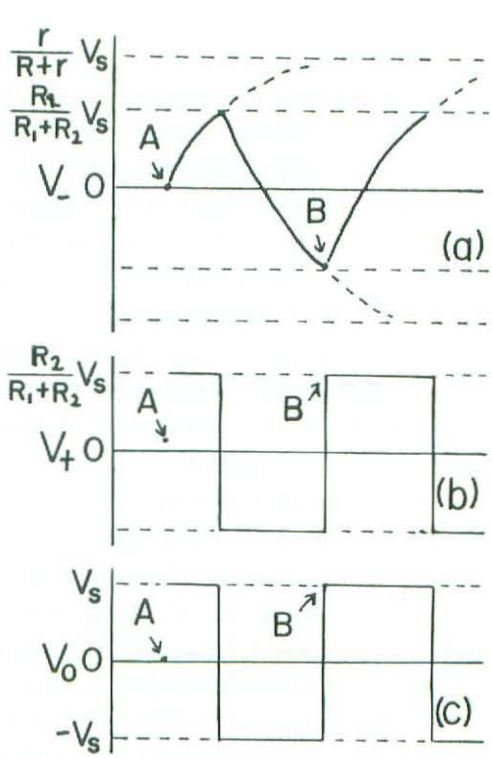

Fig. 2 Schematic diagram of the oscillation mechanism.

Temporal variations of V_-, V_+ and V_o are represented in (a), (b) and (c), respectively.

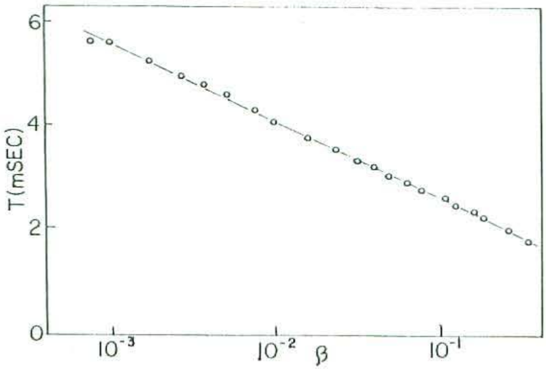

Fig. 3 β-dewendence of T.

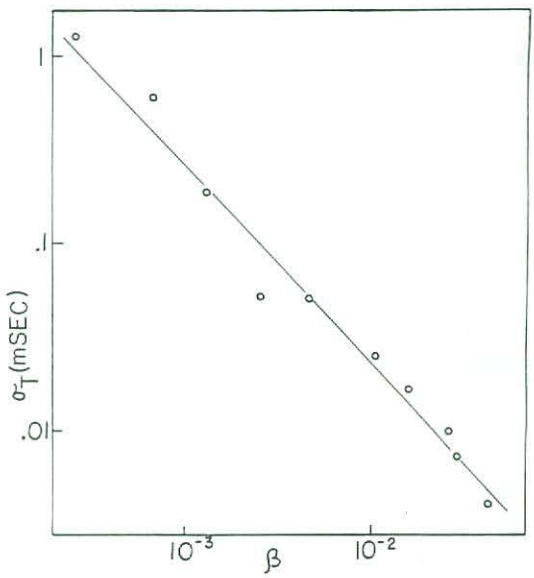

Fig. 4 β-dependence of σ_T.

There is a critical value of R, R_c (=3.94 kΩ). For $R < R_c$ the oscillation ceases. The oscillation amplitude is 15 V and the duty ratio is 0.5, and they do not vary over the range $R > R_c$. On the other hand the oscillation period T varies with R, and near R_c its fluctuation becomes remarkable. We define the controll parameter β as $\beta = (R - R_c)/R$. T is plotted against β in Fig. 3. T shows logarithmic divergence expressed as,

$$T = 1.0 - 0.67 \ln \beta \text{ (msec)}. \qquad (1)$$

To grasp the characteristics of period fluctuation we measured the standard deviation of T, σ_T by sampling 200 data of T at an interval of 1sec. The results are represented in Fig. 4. As is clear from Fig. 4 σ_T diverges according to power law, $\sigma_T \propto \beta^{-1.0}$.

2.2 Phenomenological Analyses of the Data

We define the currents and biases as shown in Fig. 1. As an initial condition we take $V_o = V_s$, $V_+ = R_2V_s/(R_1+R_2)$ and $V_- = -R_2V_s/(R_1+R_2)$ (the point B in Fig.2). V_- becomes to increase toward $RV_s/(R+r)$. From Kirchihoff's law we obtain the following equation;

$$V_- = R(I - I_1), \quad V_s - V_- = Ir,$$
$$\text{and } I_1 = C\frac{dV_-}{dt}. \qquad (2)$$

From eq. (2) the differential equation for V_- is obtained as,

$$rC\frac{dV_-}{dt} + (1 + \frac{r}{R})V_- = V_s, \qquad (3)$$

which can be solved by the variation of constant method, and gives the following solution taking into account of the initial conditions above mentioned,

$$V_- = RVs/(R+r) - \{R_2/(R_1+R_2) + R/(R+r)\}V_s\exp\{-(R+r)t/rRC\}. \qquad (4)$$

The period T is obtained by T=2t which satisfies $V_-(t) = R_2V_s/(R_1+R_2)$ where V_o turns out to be $-V_s$ from V_s. Here let us assume the presence of bias fluctuation in V_- probably due to the charge fluctuation in the condenser C to derive the fluctuation of the period. This assumption is done phenomenologically to obtain the period fluctuation, however it is considered to be not so unreasonable as discussed later. For simplicity we assume moreover that the noise bias due to the fluctuation takes two values $+\Delta V$ and $-\Delta V$. The absolute value of ΔV can be considered to nearly amount to the standard deviation of the noise. Thus the equation deciding T is expressed as,

$$\frac{RV_s}{R+r}-\left\{\frac{R_2}{R_1+R_2}+\frac{R}{R+r}\right\}V_s\times \exp\left(-\frac{R+r}{rRC}\frac{T}{2}\right)\pm\Delta V=\frac{R_2}{R_1+R_2}V_s. \quad (5)$$

At first we consider the case of $\Delta V=0$. From eq. (5) we obtain,

$$T=2rRC/(R+r)\ln\{(2RR_2+RR_1+R_2r)/(RR_1-R_2r)\}. \quad (6)$$

RR_1-R_2r should be positive, which gives the critical value of R, R_c as $R>R_c=R_2r/R_1$. R_c is estimated to be 3.90kΩ which corresponds to the experimental value 3.94kΩ. Since we observe the region of β such as $\beta<10^{-1}\ll1$ in the experiment, eq. (6) is approximated as,

$$\begin{aligned}T&\cong 2rR_cC/(r+R_c)\ln 2(1+R_2/R_1)\\&\quad-2rR_cC/(r+R_c)\ln\beta,\\&\cong 0.80-0.57\ln\beta(\mathrm{msec}),\end{aligned} \quad (7)$$

which reproduces the experimental result eq. (1) although incomplete, and explains the logarithmic divergence of T. When we take into account of ΔV we obtain two periods $T_\pm$ from eq. (5) as follows;

$$T_\pm\cong\frac{2rRC}{r+R}\times \ln\left\{\frac{2RR_2+RR_1+R_2r}{RR_1-R_2r\pm(R+r)(R_1+R_2)\Delta V/V_s}\right\}. \quad (8)$$

which is approximated for $\beta\ll1$ as,

$$T_\pm\cong\frac{2rR_cC}{r+R_c}\times \ln\left\{\frac{2(1+R_2/R_1)}{\beta\pm(1+r/R_c)(1+R_2/R_1)\Delta V/V_s}\right\}. \quad (9)$$

The standard deviation of σ_T is estimated to be nearly equal to $|T_+-T_-|$, thus we obtain,

$$\sigma_T\cong|T_+-T_-|\cong\frac{2rR_cC}{r+R_c}\times \ln\left\{\frac{\beta+(1+r/R_c)(1+R_2/R_1)\Delta V/V_s}{\beta-(1+r/R_c)(1+R_2/R_1)\Delta V/V_s}\right\}. \quad (10)$$

Although we can not estimate ΔV beforehand it should not be so large. Thus we assume here $\Delta V/V_s\ll\beta$. Under this condition eq. (10) is approximated as follows;

$$\sigma_T\cong\frac{4rR_cC}{r+R_c}\frac{\Delta V}{V_s}\left(1+\frac{r}{R_c}\right)\left(1+\frac{R_2}{R_1}\right)\beta^{-1}. \quad (11)$$

Clearly eq. (11) reproduces the experimental result $\sigma_T\propto\beta^{-1.0}$. We can estimate ΔV to be 0.5mV using eq. (11) and the result of Fig.3. The relation $\Delta V/V_s\ll\beta$ assumed before is satisfied since $V_s\cong15V$.

3. Concluding Remaks and Discussions

As mentioned in section 2 we succeeded in inducing oscillatory-nonoscillatory transition in multivibrator by adding variable resistor. The anomaly is observed in the oscillation period near the instability point. The period shows logarithmic divergence, and its fluctuation is expressed by critical index of -1 for appropriate controll parameter. These data are analyzed in a phenomenological manner by the circuit equation assuming the bias fluctuation in the condenser. Two-valued noise is assumed there for the simplicity of the analysis. This assumption is not realistic, however the extension to usual noise process such as white noise is easy and the obtained result is similar. The problem left is the unknown parameter ΔV. From what does ΔV come out? If the origin is thermal, following relation should hold,

$$(1/2)C(\Delta V)^2 \cong kT_R. \tag{12}$$

T_R is the temperature of the condenser, and k is Boltzman constant. For $T_R \cong 300K$ ΔV is estimated to be $1\mu V$ which does not coincide with the experimental value 0.5mV. Thus thermal noise of the condenser is not the origin. One of the possibilities is that input current I_{in} to the operational amplifier causes the noise. If so $\Delta V \cong I_{in} Z_{in}$, here Z_{in} is input impedance of the operational amplifier. Since $Z_{in} \cong 1M\Omega$ I_{in} is estimated to be 0.5nA. This is a probable value, however it is not clear whether I_{in} can induce the bias fluctuation in the condenser. Lastly we mention that the results obtained in this experiment are utilized in seeking wrong parts when the period of multivibrator shows anomalous elongation from the usual period.

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不安定点近傍でのマルチバイブレーターの挙動

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要旨：マルチバイブレーターにおいて外付けの抵抗を変化させることでその振動を不安定化させ，発振周期の対数的発数を誘起させた．周期の揺ぎは臨界指数－１の巾乗則で表わされた．結果は回路の時定数を決めるコンデンサーに電荷の揺ぎが発生しているという仮定を用いて解析され，実験との良い一致をみた．